Mathematical tables and graph papers are provided.

SECTION A – 65 MARKS

Question 1

In subparts (i) to (x) choose the correct option and in subparts (xi) to (xv), answer the questions as instructed.

(i) Which of the following is a homogenous differential equation? **[1]**

(a) $xydx - (x^3 - y^3)dy = 0$
(b) $(2x - 3y + 5)dy - (3x + 2y - 4)dx = 0$
(c) $y^2dx - (x^2 - xy + y^2)dy = 0$
(d) $(y^3 + 2xy)dx + 3x^2dy = 0$

(ii) The set of numbers where the function $f(x) = \frac{1}{log|x|}$ is discontinuous is: **[1]**

(a) $\{0\}$
(b) $\{1, -1\}$
(c) $\{0, 1, -1\}$
(d) None of these

(iii) The value of $\int_0^{\pi/2} logcosx\ dx$ is : [1]

(a) $-\pi log2$ (b) $-\frac{\pi}{2}\log 2$

(c) $\frac{\pi}{2}\log 2$ (d) $\pi \log 2$

(iv) If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is: [1]

(a) 1
(b) 2
(c) 0
(d) -1

(v) Assertion **(A)**: The relation R in set A of human beings in a town at a particular time given by R = {(x, y): x is exactly 5 years younger than y} is not a symmetric relation.

Reason **(R)**: A relation R on set A is said to be symmetric if (a, b) ∈ R and (b, a) ∉R [1]

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true, but R is not the correct explanation of A
(c) A is true and R is false.
(d) A is false and R s true.

(vi) If $A = tan^{-1}\frac{1}{2}$ and $B = tan^{-1}\frac{1}{3}$ then the value of $A + B$ is: [1]

(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{2}$

(vii) A and B are two independent events such that $0 < P(A) < 1$, $0 < P(B) < 1$ then which of the following is correct based on the observations (i) to (iv) [1]

(i) A and B are mutually exclusive (ii) A and B' are independent

(iii) A' and B are independent (iv) A' and B' are independent

(a) All are correct.
(b) (ii) and (iii) are only correct
(c) (i),(ii) and (iii) are only correct
(d) (ii),(iii) and (iv) are only correct

(viii) If $\int \sqrt{1 + sinx}\ f(x)\ dx = \frac{2}{3}\ (1 + sinx)^{3/2} + C$, the $f(x)$ equals: [1]

(a) $\sin x$
(b) $\tan x$
(c) $(1 + \sin x)$
(d) $\cos x$

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(ix) The function $f(x) = x^x; x > 0$ is increasing on the interval: [1]

(a) $[e, 0)$
(b) $(-\infty, \frac{1}{e}]$
(c) $[\frac{1}{e}, \infty)$
(d) $(0, e]$

(x) Ram, Laxman, Mohan and Hritvik were asked to simplify 6AB + 2(BA+AB) −4BA; where A and B are matrices of orders 3X3 and 3X3 respectively. Also, A ≠B, A≠ I, B≠ I and B≠ A^{-1}. Shown below are their responses. Only one of them gave the correct answer. Who gave the correct answer? [1]

Name	Ram	Laxman	Mohan	Hritvik
Response	6AB	8AB	10BA	8AB-2BA

(a) Ram
(b) Laxman
(c) Mohan
(d) Hritvik

(xi) If A is an invertible square matrix of order 3 and $|A| = 5$ then find the value of $|adjA|$ [1]

(xii) Consider a sample space S representing the adults in a small town who have completed the requirements for a college degree. They have been categorized according to sex and employment as follows; [1]

	Employed	Unemployed
Male	460	40
Female	140	260

An employed person is selected at random. Find the probability that chosen one is a male?

(xiii) Find the domain of the function $\sin^{-1}(4 - 3x)$ [1]

(xiv) If A and B are two sets such that n(A) =4 and n(B) =5, then the number of one-one and onto mappings from A to B is: [1]

(xv) Out of 30 consecutive integers, two are chosen at random. Find the probability of their sum being odd? [1]

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Question 2 [2]

If $x = -4$ is a root of $\Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}$, then find the other two roots using the properties of determinants.

Question 3 [2]

(i) Check the continuity for $x = 0$ of the function f defined by

$$f(x) = \begin{cases} \frac{1}{1-e^{\frac{1}{x}}} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$$

OR

(ii) If the equation of tangent to the curve $y^2 = ax^3 + b$ at point $(2, 3)$ is $y = 4x - 5$ then find the values of a and b.

Question 4 [2]

If $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) = \frac{\pi}{2}$; find the value of x.

Question 5 [2]

The equation of a curve is $3x^2 + 2xy + y^2 = 6$. Find the coordinates of the two stationary points on the curve.

Question 6 [2]

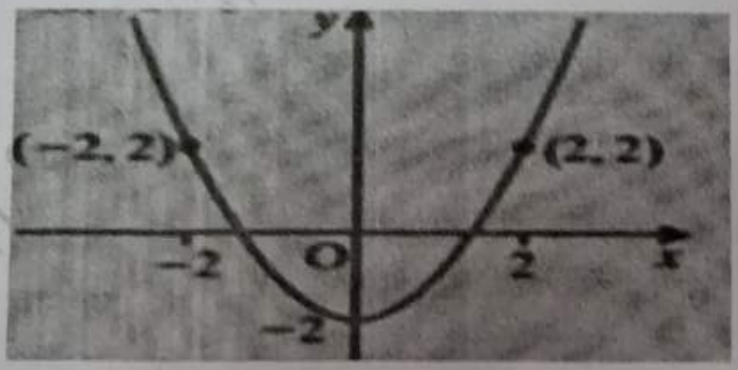
(i) Evaluate: $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$

OR

(ii) Evaluate: $\int \frac{x^2+1}{x^4+1}dx$

Question 7 [4]

The graph of the function $f : R \to R$ such that $f(x) = x^2 - 2$ is shown below.



(i) What is the range of f?

(ii) Explain why f is not an injective function?

(iii) Explain why f is not surjective?

(iv) Suggest a domain for f to make the function injective.

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Question 8 [4]

(i) The radius of the base of a cone is increasing at the rate of 3 cm / minute and the altitude is decreasing at the rate of 4 cm / minute. At what rate, the lateral surface area is changing when the radius is 7 cm and altitude is 24 cm.

OR

(ii) If $y = \cos(\sin x)$, show that $\frac{d^2y}{dx^2} + tanx \frac{dy}{dx} + y\cos^2 x = 0$

Question 9 [4]

There are three categories of students in a class of 60 students.

A: very hard-working student

B: regular but not so hard working

C: careless and irregular

10 students are in category A, 30 in category B and rest in category C. It is found that probability of students of category A, unable to get good marks in the final year examination is 0.002, of category B it is 0.02 and of category C, this probability is 0.20.A student selected at random was found to be the one who could not get good marks in the examination. Find the probability that this student is of category C.

Question 10 [4]

(i) Find the general solution of the differential equation,

$$\tan y \frac{dx}{dy} = \tan y - \frac{xtan\, y}{y} - x$$

OR

(ii) Solve: $\cos(x + y)\, dy = dx$, given $y(0) = 0$

Question 11 [6]

Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible when revolved about one of its sides. Also find the maximum sweep out volume in terms of π

Question 12 [6]

(i) Evaluate $\int \frac{2}{(1-x)(x^2+1)}\, dx$

OR

(ii) Evaluate $\int_0^{\pi/2} \frac{sin^2 x}{1+sinx\, cosx}\, dx$

Question 13 [6]

(i) An amount of Rs 5000 is put into three investments at the rate of 6 %, 7 %, and 8 % per annum respectively. The total annual income is Rs 358.If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment by matrix method.

OR

(ii) $B.A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & -1 \\ -3 & 2 & -4 \end{bmatrix}$. Find A^{-1} and hence solve the system of linear equations

$x + 2y - 3z = -4$, $2x + 5y + 2z = 2$, $3x - 5y - 4z = 11$.

Question 14 [6]

Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denotes the larger of the two numbers obtained. Find the probability distribution of X and also find the expectation of X.

SECTION B – 12 MARKS

Question 15 [12]

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v) answer the questions as instructed.

(i) If for two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then the angle between them is

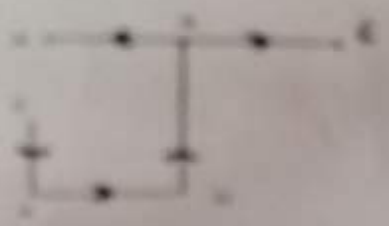
(a) 45°
(b) 90°
(c) 0°
(d) 60°

(ii) If the x-coordinate of a point on the line joining (1, 1, 1) and (5, 1, -1) is 4, then its z-coordinate is

(a) 4/3
(b) 3
(c) 1/3
(d) -1

(iii) In the given figure, $AB = BC = EF = \frac{1}{2}DE$

Statement 1: $\overrightarrow{EF}$ and $\overrightarrow{BC}$ are like vectors.
Statement 2: $\overrightarrow{EB}$ and $\overrightarrow{BC}$ are equal vectors.



(a) Only statement 1
(b) Only statement 2
(c) Both statements 1 and 2
(d) Neither statement 1 nor statement 2

(iv) The area of the parallelogram whose adjacent sides are $\hat{i} + \hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$ is :

(v) Find the equation of the plane passing through (1, 3, 4) and making equal intercepts on the coordinate axes.

Question 16 [2]

(i) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$ and $\vec{a}.\vec{b} = 1$, then find the vector $\vec{b}$

OR

(i) If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{j} + 4\hat{k}$, then find the vector form of the component of $\vec{a}$ along $\vec{b}$

Question 17 [4]

(i) Find the foot of perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$

OR

(ii) Find the distance of the point (1, -2, 3) from the plane $x - y + z = 5$ measured along a line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

Question 18 [4]

Using integration, find the area of the region bounded by the line $y = 3x + 2$, the x axis and the ordinates $x = -2$ *and* $x = 1$

SECTION C- 15 MARKS

Question 19 [5]

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) The slope of the regression line of Y on X is also referred to as
(a) Regression coefficient of X on Y
(b) The correlation coefficient of X on Y
(c) Regression coefficient of Y on X
(d) The correlation coefficient of Y on X

(ii) Assertion (A): If the cost function of a firm is given by $C(x) = 2x^2 + x - 4$ then the average cost is given by $2x + 1$

Reason (R): Average cost, $AC(x) = \frac{C}{x}$

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true, but R is not the correct explanation of A
(c) A is true and R is false.
(d) A is false and R is true

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(iii) At a toy factory, the production cost per day for x number of toys is given by $C(x) = 0.001x^3 - 0.01\,x^2 - 20\,x + 3500$. Which of these represents the approximate additional cost to produce 201^{st} toy?

(a) C (200)
(b) MC (200)
(c) AC (200)
(d) MC (201)

(iv) If $\bar{x} = 40, \bar{y} = 6,\ \sigma_x = 10,\ \sigma_y = 1.5$ and $r = 0.9$ for the two sets of data X and Y, then regression line of X on Y will be:

(v) A company sells x packets of biscuits each day at RS 20 per packet. The cost of manufacturing these packets is Rs 8 per packet plus a fixed daily overhead cost of Rs 600.What will be the profit function?

Question 20 [2]

(i) The demand function for a certain product is represented by the function $p = 200 + 20x - x^2$ (in Rs), where x the number of units is demanded and p is the price per unit. What is the exact amount of increase in revenue when sales are increased from 10 to 11?

OR

(ii) The demand function for a product is given by $p = \frac{5}{x+3}$. Show that the marginal revenue function is a decreasing function.

Question 21 [4]

The Personnel Manager of a factory wants to find a measure which he can use to fix the monthly income of persons applying for a job in production department. As an experiment project, he collected data on 7 persons from that department referring to years of service and their monthly income.

Years of service (x)	11	7	9	5	8	6	10
Income in 00 Rs (y)	10	8	6	5	9	7	11

What initial start (y) would you recommend for a person applying for a job after having served in similar capacity in another factory for 13 years (x).

Question 22 [4]

(i) An oil company requires 12,000, 20,000 and 15,000 barrels of high -grade, medium- grade and low-grade oil, respectively. Refinery A produces 100, 300 and 200 barrels per day of high-grade, medium-grade and low-grade oil, respectively, while refinery B produces 200, 400 and 100 barrels per day of high – grade, medium- grade and low-grade oil, respectively. If refinery A costs ₹400 per day and refinery B costs ₹300 per day to operate, how many days should each be run to minimize costs while satisfying requirements.

OR

(ii) Solve the Linear Programing Problem graphically.

Maximize $= 5x + 2y$, subject to constraints

$$x - 2y \leq 2$$
$$3x + 2y \leq 12$$
$$-3x + 2y \leq 3$$
$$x \geq 0$$
$$y \geq 0$$

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